

NAG C Library Function Document

nag_complex_polygamma (s14afc)

1 Purpose

nag_complex_polygamma (s14afc) returns the value of the k th derivative of the psi function $\psi(z)$, for complex z and $k = 0, 1, \dots, 4$.

2 Specification

Complex nag_complex_polygamma(Complex z, Integer k, NagError *fail)

3 Description

This routine evaluates an approximation to the k th derivative of the psi function $\psi(z)$ given by

$$\psi^{(k)}(z) = \frac{d^k}{dz^k} \psi(z) = \frac{d^k}{dz^k} \left(\frac{d}{dz} \log_e \Gamma(z) \right),$$

where $z = x + iy$ is complex provided $y \neq 0$ and $k = 0, 1, \dots, 4$. If $y = 0$, z is real and thus $\psi^{(k)}(z)$ is singular when $z = 0, -1, -2, \dots$.

Note that $\psi^{(k)}(z)$ is also known as the *polygamma* function. Specifically, $\psi^{(0)}(z)$ is often referred to as the *digamma* function and $\psi^{(1)}(z)$ as the *trigamma* function in the literature. Further details can be found in Abramowitz and Stegun (1972).

nag_complex_polygamma is based on a modification of the method proposed by Kölbig K S (1972).

To obtain the value of $\psi^{(k)}(z)$ when z is real, nag_real_polygamma (s14aec) can be used.

4 Parameters

- | | | |
|----|--|---------------------|
| 1: | z – Complex | <i>Input</i> |
| | <i>On entry:</i> the argument z of the function. | |
| | <i>Constraint:</i> z.re must not be ‘too close’ (see Section 5) to a non-positive integer when z.im = 0.0. | |
| 2: | k – Integer | <i>Input</i> |
| | <i>On entry:</i> the function $\psi^{(k)}(z)$ to be evaluated. | |
| | <i>Constraint:</i> $0 \leq \mathbf{k} \leq 4$. | |
| 3: | fail – NagError * | <i>Input/Output</i> |
| | The NAG error parameter (see the Essential Introduction). | |

5 Error Indicators and Warnings

NE_INT

On entry, **k** = <value>.

Constraint: $0 \leq \mathbf{k} \leq 4$.

NE_COMPLEX

On entry, **z** = (<value>, <value>).

Constraint: **z.re** must not be ‘too close’ to a non-positive integer when **z.im** = 0.0. That is, $|\mathbf{z.re} - \text{nint}(\mathbf{z.re})| \geq \text{machine precision} \times |\text{nint}(\mathbf{z.re})|$

NE_OVERFLOW_LIKELY

The evaluation has been abandoned due to the likelihood of overflow. The result is returned as zero.

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please consult NAG for assistance.

6 Further Comments**6.1 Accuracy**

Empirical tests have shown that the maximum relative error is a loss of approximately two decimal places of precision.

6.2 References

Kölbig K S (1972) Programs for computing the logarithm of the gamma function, and the digamma function, for complex arguments *Comp. Phys. Comm.* **4** 221–226

Abramowitz M and Stegun I A (1972) *Handbook of Mathematical Functions* Dover Publications (3rd Edition)

7 See Also

None.

8 Example

The example program evaluates the psi (trigamma) function $\psi^{(1)}(z)$ at $z = -1.5 + 2.5i$, and prints the results.

8.1 Program Text

```
/* nag_complex_polygamma (s14afc) Example Program.
 *
 * Copyright 2000 Numerical Algorithms Group.
 *
 * NAG C Library
 *
 * Mark 6, 2000.
 */
#include <stdio.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <nags.h>

int main(void)
{
    Complex z, z__1;
    Integer exit_status=0;
    Integer k;
    NagError fail;

    INIT_FAIL(fail);
    Vprintf("s14afc Example Program Results\n\n");
    /* Skip heading in data file */
    Vscanf("%*[^\\n] ");
```

```

Vprintf("          z          k          (D^K/DZ^K)psi(z)\n\n");
while(scanf(" (%lf,%lf)%ld%*[\n] ", &z.re, &z.im, &k) != EOF)
{
    z__1 = s14afc (z, k, &fail);
    if (fail.code == NE_NOERROR)
        Vprintf("(%5.1f, %5.1f) %6ld (%12.4e, %12.4e)\n",
                z.re, z.im, k, z__1.re, z__1.im);
    else
    {
        Vprintf("Error from s14afc.\n%s\n", fail.message);
        exit_status = 1;
        goto END;
    }
}
END:
return exit_status;
}

```

8.2 Program Data

s14afc Example Program Data

(1.2,5.0)	0
(0.5,-0.2)	1
(-1.5,2.5)	1
(8.0,3.3)	3
(2.9,7.5)	4 : Values of z and k

8.3 Program Results

s14afc Example Program Results

z	k	(D^K/DZ^K)psi(z)
(1.2, 5.0)	0	(1.6176e+00, 1.4312e+00)
(0.5, -0.2)	1	(3.4044e+00, 2.5394e+00)
(-1.5, 2.5)	1	(-1.9737e-01, -2.4271e-01)
(8.0, 3.3)	3	(1.1814e-03, -3.4188e-03)
(2.9, 7.5)	4	(-5.0227e-04, -1.4955e-03)
